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Robust D-stability analysis for linear uncertain discrete singular systems with state delay

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Abstract

In this work, by using the maximum modulus principle and the spectral radii of matrices, a new robust D-stability (i.e., robust eigenvalue clustering in a specified circular region) condition is proposed to ensure that, for all admissible structured parameter uncertainties, the linear discrete singular system with state delay is regular, causal and D-stable. The proposed criterion is mathematically proved to be less conservative than the existing one reported very recently.

Keywords: D-stability robustness; Discrete singular systems; Structured parameter uncertainties; State delay

1. Introduction

Linear discrete singular delay systems can be found in many practical applications, such as electrical networks, large-scale systems, constrained robots, and economical systems [1]. On the other hand, to ensure both stability robustness and a certain performance robustness, it is important to guarantee that the eigenvalues of a linear time-invariant system under parameter perturbations remain in a specified region. Therefore, very recently, Xu and his associates [2,3] studied the robust D-stability (i.e., robust

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eigenvalue clustering in a specified circular region) problem of a linear discrete singular delay system with structured (elemental) parameter uncertainties. Here it should be emphasized that the robust D-stability analysis of linear uncertain discrete singular delay systems should consider not only the D-stability robustness but also system regularity and causality simultaneously. Under the assumption that the linear discrete nominal singular system, $Ex(k+1) = Ax(k)$ denoted by (E, A), is regular, causal and D-stable, by using the pulse-response sequence matrix approach of Chou [4], Xu and Lam [3] presented the robust D-stability condition, which is suitable for more general cases than that previously considered by Xu et al. [2], to guarantee the D-stability robustness of the following linear discrete singular time-delay system denoted by $(E, A + \Delta\tilde{A})$ with structured parameter uncertainties:

$$Ex(k+1) = (A + \Delta A)x(k) + (A_d + \Delta A_d)x(k-\tau), \qquad Ex(0) = Ex_0, \tag{1}$$

where $E \in R^{n\times n}$, $A \in R^{n\times n}$, $A_d \in R^{n\times n}$, $x(k) \in R^n$, ΔA and ΔA_d denote the $n \times n$ time-invariant structured (elemental) parameter uncertain matrices, and $\tau \geq 1$ is a known positive integer representing the time delay. Here the matrix E may be a singular matrix with $\text{rank}(E) \leq n$. In many applications, the matrix E is a structural information matrix rather than a parameter matrix, i.e., the elements of E contain only structural information regarding the problem considered. Suppose the uncertain matrices ΔA and ΔA_d are bounded by the following inequalities:

$$|\Delta A| \leq U \qquad \text{and} \qquad |\Delta A_d| \leq U_d, \tag{2}$$

where U and U_d are given nonnegative constant matrices and represent highly structured information.

The purpose of this work is, by using the maximum modulus principle [5] and the spectral radii of matrices, to propose another new approach for studying the robust D-stability problem of the linear discrete uncertain singular delay system $(E, A + \Delta\tilde{A})$ in (1) under the same assumption as in the work by Xu and Lam [3]. This work is organized as follows. The main result is presented in Section 2. By mathematical analysis, a comparison between the degree of conservativism of the two sets of sufficient conditions which are, respectively, proposed in this work and by Xu and Lam [3] is given in Section 3. A numerical example is also given for illustration in this section. Finally, Section 4 offers some conclusions.

2. Main result

In this work, as in the work by Xu and Lam [3], we consider the linear discrete uncertain singular delay system $(E, A + \Delta\tilde{A})$ in (1) under the assumption that the linear discrete nominal singular system (E, A) is regular, causal and D-stable. Our problem is to determine the condition such that, under the aforementioned assumption, the linear discrete uncertain singular delay system $(E, A + \Delta\tilde{A})$ is still regular, causal and D-stable.

Before we analyze the D-stability robustness of the linear discrete uncertain singular delay system $(E, A + \Delta\tilde{A})$, the following definition and lemmas need to be introduced.

Definition (*[6,7]*). The linear discrete singular system $Ex(k+1) = Ax(k)$ is termed regular and causal if $\det(zE - A)$ is not identically zero and if $\text{rank}(E) =$ degree of $\det(zE - A)$ in the z-plane, where $E \in R^{n\times n}$, $A \in R^{n\times n}$, $x(k) \in R^n$, and $\text{rank}(E) \leq n$.

Lemma 1 (*[6,7]*). *The linear discrete singular system* (E, A) *is said to be stable, regular and causal if and only if the following two conditions are satisfied:* (i) *all the eigenvalues of* $\det(zE - A) = 0$ *lie inside the unit circle of the z-plane, and* (ii) $(zE - A)^{-1}$ *is proper.*

Lemma 2. *If all the finite eigenvalues of the linear discrete nominal singular system* (E, A) *lie inside a specified circular region* $D(e, f)$ *centered at* $(e, 0)$ *with radius* f *(i.e., the linear discrete nominal singular system* (E, A) *is* $D(e, f)$*-stable), where* $f > 0$ *and* $|e| + f < 1$*, then* $\det(zE - A) \neq 0$*, for* $|(z - e)/f| \geq 1$.

Proof. If all the finite eigenvalues of the nominal singular system (E, A) lie inside a specified circular region $D(e, f)$, this implies that all the finite eigenvalues of the nominal singular system (E, A) will never be on the boundary or the outside of the specified circular region $D(e, f)$. So we have the stated results. □

Lemma 3. *If the pair* (E, A) *is regular and causal, and* $D(e, f)$*-stable, then the following two conditions are satisfied:* (i) $(zE - A)^{-1}$ *is proper, and* (ii) $\det(vE - \bar{A}) \neq 0$*, for* $|v| \geq 1$*, where* $v = (z - e)/f$ *and* $\bar{A} = (A - eE)/f$.

Proof. Let $v = (z - e)/f$; from Lemmas 1 and 2, we have the stated results. □

Lemma 4. *The linear discrete uncertain singular delay system* $(E, A + \Delta\tilde{A})$ *in* (1) *is regular and causal if and only if the pair* $(E, A + \Delta A)$ *which denotes* $Ex(k + 1) = (A + \Delta A)x(k)$ *is regular and causal.*

Proof. This lemma can be directly obtained by following the same proof procedure as given by Xu et al. [2]. □

Lemma 5. *All the finite eigenvalues of the linear discrete uncertain singular delay system* $(E, A + \Delta\tilde{A})$ *in* (1) *lie inside the specified circular region* $D(e, f)$*, if*

$$\det(zE - A - \Delta A - z^{-\tau}(A_d + \Delta A_d)) \neq 0, \qquad \text{for } |(z - e)/f| \geq 1. \tag{3}$$

Proof. If $\det(zE - A - \Delta A - z^{-\tau}(A_d + \Delta A_d)) \neq 0$, for $|(z - e)/f| \geq 1$, this implies that all the finite eigenvalues of the linear discrete uncertain singular delay system (1) will not be on the boundary or the outside of the specified circular region $D(e, f)$. So, we have the stated results. □

Lemma 6 ([8]). *For a matrix* $W \in C^{n\times n}$*, if* $r[W] < 1$*, then* $\det(I \pm W) \neq 0$*, where* $r[W]$ *denotes the spectral radius of the matrix* W.

Lemma 7 ([9]). *For any* $n \times n$ *constant complex matrices* U*,* V *and* W *with* $|U| \leq W$*, where* $|U|$ *denotes the modulus matrix of the matrix* U*, we have*

(i) $|UV| \leq |U||V| \leq W|V|$;
(ii) $|U + V| \leq |U| + |V| \leq W + |V|$;
(iii) $r[U] \leq r[|U|] \leq r[W]$;
(iv) $r[UV] \leq r[|U||V|] \leq r[W|V|]$;
(v) $r[U + V] \leq r[|U| + |V|] \leq r[W + |V|]$.

Lemma 8 ([5]). *If a function* $f(z)$ *is analytic in a simple closed region* Ω *and not constant in the interior* Ω*, then the maximum value of* $|f(z)|$*, for* $z \in \Omega$*, must occur on the boundary of* Ω.

Lemma 9 ([10]). *Let* $\tilde{G}(z)$ *be a square rational matrix and be decomposed uniquely as* $\tilde{G}(z) = \tilde{G}_{sp}(z) + \tilde{G}_p(z)$*, where* $\tilde{G}_{sp}(z)$ *is a strictly proper rational matrix and* $\tilde{G}_p(z)$ *is a polynomial matrix. Then* $\tilde{G}^{-1}(z)$ *is proper if and only if* $\tilde{G}_p^{-1}(z)$ *exists and is proper.*

With the preceding lemmas, we can analyze the D-stability robustness of the linear discrete uncertain singular delay systems $(E, A + \Delta\tilde{A})$ in (1). For the linear discrete uncertain singular delay system $(E, A + \Delta\tilde{A})$, we assume that the linear discrete nominal singular system (E, A) is regular, causal and D-stable; then, from Lemma 1, we can see that $(zE - A)^{-1}$ is a proper rational matrix. Because $(zE - A)^{-1}$ is a proper rational matrix, it can be uniquely decomposed as [7,11]

$$(zE - A)^{-1} = J + G_{sp}(z), \tag{4}$$

where $G_{sp}(z)$ is a strictly proper matrix part of $(zE - A)^{-1}$ and J is a constant matrix part of $(zE - A)^{-1}$.

In the following, we present a theorem for ensuring the regularity, causality and D-stability of the linear discrete uncertain singular delay system $(E, A + \Delta\tilde{A})$ in (1).

Theorem. *Assume that the linear discrete nominal singular system (E, A) is regular and causal, and assume that the linear discrete nominal singular system (E, A) has all its finite eigenvalues located inside a specified circular region $D(e, f)$. The linear discrete uncertain singular delay system $(E, A + \Delta\tilde{A})$ in (1) is still regular and casual, and has all its finite eigenvalues retained within the same specified circular region as the linear discrete nominal singular system (E, A) does, if the following inequalities are satisfied:*

$$r[|J|U] < 1, \tag{5a}$$

and

$$r[f^{-1}\tilde{H}(U + \gamma^{-\tau}(|A_d| + U_d))] < 1, \tag{5b}$$

where the constant matrix J is given in Eq. (4); $\tilde{H} = [\tilde{h}_{ik}]_{n\times n} = [\max|\tilde{h}_{ik}(j\theta)|]_{n\times n}$ and $\gamma = \min|e + f\exp(j\theta)|$, for $\theta \in [0, 2\pi]$; $\tilde{h}_{ik}(j\theta)$ is the ik-th element of $(\exp(j\theta)E - \bar{A})^{-1}$ with $\bar{A} = (A - eE)/f$; $j = \sqrt{-1}$.

Proof. Firstly, we show that the linear discrete uncertain singular time-delay system $(E, A + \Delta\tilde{A})$ in (1) is regular and casual. Because the linear discrete nominal singular system (E, A) is regular and casual, then from Lemma 1, we have that $(zE - A)^{-1}$ is proper. Using Eq. (4), we can express $(zE - A - \Delta A)^{-1}$ as

$$\begin{aligned}(zE - A - \Delta A)^{-1} &= (I - (zE - A)^{-1}\Delta A)^{-1}(zE - A)^{-1}\\ &= (I - J\Delta A - G_{sp}(z)\Delta A)^{-1}(zE - A)^{-1}.\end{aligned} \tag{6}$$

By Lemma 9, the nonsingularity of $(I - J\Delta A)$ implies that $(I - J\Delta A - G_{sp}(z)\Delta A)^{-1}$ is proper, or equivalently $(zE - A - \Delta A)^{-1}$ is proper.

From Lemma 7, and using Eq. (2), we can obtain

$$r[J\Delta A] \le r[|J||\Delta A|] \le r[|J|U]. \tag{7}$$

If the inequality in Eq. (5a) is satisfied, then, from Eq. (7), we have that $r[J\Delta A] < 1$. That is, if the inequality in Eq. (5a) is satisfied, then $(I - J\Delta A)^{-1}$ is nonsingular. Therefore, the inequality in Eq. (5a) ensures that $(zE - A - \Delta A)^{-1}$ is proper. Thus, from Lemmas 1 and 4, it can be seen that the linear discrete uncertain singular delay system $(E, A + \Delta\tilde{A})$ is regular and causal.

Next, we will show that the linear discrete uncertain singular delay system $(E, A + \Delta\tilde{A})$ has all its finite eigenvalues retained in the specified circular region $D(e, f)$. Since all the finite eigenvalues of the linear discrete nominal singular system (E, A) lie inside the specified circular region $D(e, f)$, then, from Lemma 3, we have

$$\det(vE - \bar{A}) \neq 0, \qquad \text{for } |v| \geq 1, \tag{8}$$

where $v = (z - e)/f$ (i.e., $z = fv + e$) and $\bar{A} = (A - eE)/f$. If the following inequality holds:

$$\det\left(I - (vE - \bar{A})^{-1}\left(\frac{\Delta A}{f} + \frac{(A_d + \Delta A_d)}{f(fv+e)^{\tau}}\right)\right) \neq 0, \qquad \text{for } |v| \geq 1, \tag{9}$$

then, from Eqs. (8) and (9), we can get

$$\begin{aligned} &\det(zE - A - (\Delta A + z^{-\tau}(A_d + \Delta A_d))) \\ &\quad = f \bullet \det\left(vE - \bar{A} - \left(\frac{\Delta A}{f} + \frac{(A_d + \Delta A_d)}{f(fv+e)^{\tau}}\right)\right) \\ &\quad = f \bullet \det(vE - \bar{A}) \bullet \det\left(I - (vE - \bar{A})^{-1}\left(\frac{\Delta A}{f} + \frac{(A_d + \Delta A_d)}{f(fv+e)^{\tau}}\right)\right) \neq 0, \end{aligned} \tag{10}$$

for $|v| \geq 1$. Hence, from Eq. (10) and Lemma 5, we can obtain that all the finite eigenvalues of the system $(E, A + \Delta\tilde{A})$ lie inside the specified circular region $D(e, f)$. That is, if the inequality in (9) holds, then, from the above-mentioned statements, we can conclude that the linear discrete uncertain singular delay system $(E, A + \Delta\tilde{A})$ has all its finite eigenvalues retained in the specified circular region $D(e, f)$. Therefore, in the following, we will derive the condition under which the inequality in (9) holds. Let $\zeta = v^{-1}$; then Eq. (9) becomes

$$\det\left(I - (\zeta^{-1}E - \bar{A})^{-1}\left(\frac{\Delta A}{f} + \frac{(A_d + \Delta A_d)}{f(f\zeta^{-1}+e)^{\tau}}\right)\right) \neq 0, \qquad \text{for } |\zeta| \leq 1. \tag{11}$$

From Eq. (8), we can see that $(vE - \bar{A})^{-1}$ is analytic in the domain $|v| \geq 1$, so we can conclude that $(\zeta^{-1}E - \bar{A})^{-1}$ is analytic in the bounded domain $|\zeta| \leq 1$. Besides this, since the multiple roots of $(\frac{\Delta A}{f} + \frac{(A_d+\Delta A_d)}{f(f\zeta^{-1}+e)^{\tau}})$ are at $\zeta = -\frac{f}{e}$ with $|e| < 1$, there are no roots lying inside the bounded domain $|\zeta| \leq 1$ [12]. Hence, $(\frac{\Delta A}{f} + \frac{(A_d+\Delta A_d)}{f(f\zeta^{-1}+e)^{\tau}})$ is also analytic in the bounded domain $|\zeta| \leq 1$. Therefore, $(\zeta^{-1}E - \bar{A})^{-1}(\frac{\Delta A}{f} + \frac{(A_d+\Delta A_d)}{f(f\zeta^{-1}+e)^{\tau}})$ is analytic and not constant in the bounded domain $|\zeta| \leq 1$. Thus, from Lemma 8, the maximum value of $\left|(\zeta^{-1}E - \bar{A}_9)^{-1}(\frac{\Delta A}{f} + \frac{(A_d+\Delta A_d)}{f(f\zeta^{-1}+e)^{\tau}})\right|$ will occur on the boundary of the domain $|\zeta| \leq 1$. Let $\tilde{h}_{ik}$ denote the maximum value of $|\tilde{h}_{ik}(j\theta)|$ for $\theta \in [0, 2\pi]$, where $\tilde{h}_{ik}(j\theta)$ is the ik-th element of $(\exp(j\theta)E - \bar{A})^{-1}$. From the above-mentioned results, Lemma 7 and the definition of $\tilde{h}_{ik}$, and using Eqs. (2) and (5b), we can obtain

$$\begin{aligned} &r\left[(\zeta^{-1}E - \bar{A})^{-1}\left(\frac{\Delta A}{f} + \frac{(A_d + \Delta A_d)}{f(f\zeta^{-1}+e)^{\tau}}\right)\right] \leq r\left[\left|(\zeta^{-1}E - \bar{A})^{-1}\left(\frac{\Delta A}{f} + \frac{(A_d + \Delta A_d)}{f(f\zeta^{-1}+e)^{\tau}}\right)\right|\right] \\ &\quad \leq r\left[\left|(\exp(j\theta)E - \bar{A})^{-1}\left(\frac{\Delta A}{f} + \frac{(A_d + \Delta A_d)}{f(f\exp(j\theta)+e)^{\tau}}\right)\right|\right] \\ &\quad \leq r\left[|(\exp(j\theta)E - \bar{A})^{-1}|\left(\left|\frac{\Delta A}{f}\right| + \frac{(|A_d| + |\Delta A_d|)}{f(|f\exp(j\theta)+e|^{\tau})}\right)\right] \\ &\quad \leq r\left[[|(\tilde{h}_{ik}(j\theta))|]_{n\times n}\left(\frac{U}{f} + \frac{(|A_d| + U_d)}{f\gamma^{\tau}}\right)\right] \end{aligned}$$

$$\le r\left[[\tilde{h}_{ik}]_{n\times n}\left(\frac{U}{f}+\frac{(|A_d|+U_d)}{f\gamma^{\tau}}\right)\right]$$
$$= r[f^{-1}\tilde{H}(U+\gamma^{-\tau}(|A_d|+U_d))]$$
$$< 1, \qquad \text{for } \theta\in[0,2\pi] \text{ and } |\zeta|\le 1. \tag{12}$$

Following Lemma 6, the inequality (12) implies Eq. (11). That is, the inequality in (5b) guarantees that the inequality in (9) holds. Therefore, we have proved that, if both the inequalities in Eqs. (5a) and (5b) are satisfied, then the linear discrete uncertain singular delay system $(E, A+\Delta\tilde{A})$ is still regular and causal, and has all its finite eigenvalues retained inside the specified circular region $D(e, f)$. Thus the proof is completed. □

Remark. The sufficient conditions (5a) and (5b) can give the explicit relationship of the allowable bounds of a linear uncertain discrete singular system with state delay. The sufficient condition (5a) can be used to guarantee that the linear uncertain discrete singular time-delay system is regular and causal, and the sufficient condition (5b) can be used to guarantee that all the finite eigenvalues of the linear uncertain discrete singular time-delay system are retained in the specified circular region.

3. Mathematical comparison of proposed criterion and Xu–Lam's criterion

In this section, by using mathematical analysis, we will compare the sufficient condition proposed in this work with that given by Xu and Lam [3].

Xu and Lam [3] have shown that the linear discrete uncertain singular time system (1) is still regular and causal, and has all its finite eigenvalues retained within the same specified circular region $D(e, f)$ as the linear discrete nominal singular system (E, A) does, if the following inequality holds:

$$r[f^{-1}\tilde{T}(U+\delta^{-\tau}(|A_d|+U_d))] < 1, \tag{13}$$

where $\delta = \min\{|f+e|, |f-e|\}$, and $\tilde{T}$ is defined by

$$\tilde{T} = [\tilde{t}_{ik}]_{n\times n} = \left[\sum_{k=0}^{\infty}|\tilde{Y}(k)|\right]_{n\times n}, \tag{14}$$

in which $\tilde{Y}(k)$ is the pulse-response sequence matrix of $G(v)$ defined as

$$G(v) = (vE-\bar{A})^{-1} = \sum_{k=0}^{\infty}\tilde{Y}(k)v^{-k} \tag{15a}$$

with $\bar{A} = (A-eE)/f$. Eq. (15a) can be rewritten as

$$G(v) = (vE-\bar{A})^{-1} = \left(\frac{(z-e)}{f}E - \frac{(A-eE)}{f}\right)^{-1} = f(zE-A)^{-1} = f(J+G_{sp}(z)), \tag{15b}$$

and

$$G(v) = \sum_{k=0}^{\infty}\tilde{Y}(k)v^{-k}$$
$$= \sum_{k=0}^{\infty}\tilde{Y}(k)f^{k}(z-e)^{-k}$$

$$
\begin{aligned}
&= \sum_{k=0}^{\infty} \tilde{Y}(k) f^k \left[z^{-k} + kez^{-k-1} + \frac{k(k+1)}{2} e^2 z^{-k-2} + \frac{k(k+1)(k+2)}{6} e^3 z^{-k-3} + \cdots \right] \\
&= \tilde{Y}(0) \\
&\quad + \sum_{k=1}^{\infty} \tilde{Y}(k) f^k \left[z^{-k} + kez^{-k-1} + \frac{k(k+1)}{2} e^2 z^{-k-2} + \frac{k(k+1)(k+2)}{6} e^3 z^{-k-3} + \cdots \right] \\
&= \tilde{Y}(0) + \tilde{G}_{sp}(z).
\end{aligned}
\tag{15c}
$$

Now, we compare the proposed sufficient condition in Eqs. (5a) and (5b) with the sufficient condition (13) given by Xu and Lam [3]. From Eqs. (14), (15b) and (15c), we can get

$$
|J| = |f^{-1}\tilde{Y}(0)| \le f^{-1} \left[\sum_{k=0}^{\infty} |\tilde{Y}(k)| \right]_{n\times n} = f^{-1}\tilde{T},
\tag{16a}
$$

and

$$
\begin{aligned}
\tilde{H} &= [\max |\tilde{h}_{ik}(j\theta)|]_{n\times n} = [\max |(\exp(j\theta)E - \bar{A})^{-1}|]_{n\times n} \\
&= \left[\max \left| \sum_{k=0}^{\infty} \tilde{Y}(k) \exp(-jk\theta) \right| \right]_{n\times n} \\
&\le \left[\sum_{k=0}^{\infty} \left| \tilde{Y}(k) \right| \right]_{n\times n} = \tilde{T}, \qquad \text{for } \theta \in [0, 2\pi].
\end{aligned}
\tag{16b}
$$

Because

$$
\gamma = \min |e + f \exp(j\theta)| = \min\{|f + e|, |f - e|\} = \delta,
\tag{17}
$$

then, from Eqs. (16a), (16b) and (17), and from Lemma 7, we can obtain

$$
r[|J|U] \le r[f^{-1}\tilde{T}(U + \delta^{-\tau}(|A_d| + U_d))],
$$

and

$$
r[f^{-1}\tilde{H}(U + \gamma^{-\tau}(|A_d| + U_d))] \le r[f^{-1}\tilde{T}(U + \delta^{-\tau}(|A_d| + U_d))].
$$

This proves mathematically that the sufficient condition in (5a) and (5b) proposed in this work is less conservative than the sufficient condition (13) proposed by Xu and Lam [3].

In what follows, a numerical example is given for illustration.

Example. Consider a linear discrete singular time-delay system with structured (elemental) parameter uncertainties described as

$$
Ex(k+1) = Ax(k) + \Delta Ax(k) + A_d x(k-1) + \Delta A_d x(k-1),
\tag{18}
$$

where

$$
E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \qquad A = \begin{bmatrix} 0 & 0.8 & 0 \\ 3.2 & 2.4 & -1 \\ -0.4 & 0.4 & 0 \end{bmatrix}, \qquad A_d = \begin{bmatrix} 0 & 0 & 0 \\ 0.1 & 0.1 & 0.1 \\ 0.03 & 0.05 & 0 \end{bmatrix},
$$

$$|\Delta A| \le \begin{bmatrix} 0.01 & 0.01 & 0 \\ 0.01 & 0.01 & 0.01 \\ 0.01 & 0.01 & 0.01 \end{bmatrix}, \qquad \text{and} \qquad |\Delta A_d| \le \begin{bmatrix} 0.01 & 0.01 & 0 \\ 0.01 & 0.01 & 0.01 \\ 0 & 0.01 & 0.01 \end{bmatrix}.$$

All the finite eigenvalues of the linear discrete nominal singular system (E, A) in Eq. (18) are $0.2 \pm \tilde{j}0.5292$ which are located inside a specified circular region $D(0.2, 0.8)$ centered at $0.2 + \tilde{j}0$ with radius 0.8.

By using the sufficient condition in (13) proposed by Xu and Lam [3], we have

$$r[f^{-1}\tilde{T}(U + \delta^{-1}(|A_d| + U_d))] = 1.2116 \nless 1.$$

Then, no conclusion can be drawn. That is, the sufficient condition of Xu and Lam [3] cannot be applied in this example.

Now, applying the sufficient condition in (5), we have

$$r[|J|U] = 0.01 < 1,$$

and

$$r[f^{-1}\tilde{H}(U + \gamma^{-1}(|A_d| + U_d))] = 0.9894 < 1.$$

Therefore, the proposed sufficient condition in (5) is satisfied. This implies that the linear discrete uncertain singular time-delay system (18) is still regular and causal, and has all its finite eigenvalues retained within the same circular region as the linear discrete nominal singular system (E, A) does. It is obvious that the proposed sufficient condition (5) can overcome the conservatism of the sufficient condition (13) given by Xu and Lam [3]. Note that this fact that the proposed sufficient condition (5) is less conservative than that given by Xu and Lam [3] has been proved by mathematical analysis in this section.

4. Conclusions

Under the assumptions that the linear discrete singular time-delay system is regular and causal, and has all its finite eigenvalues lying inside a specified circular region, a new sufficient condition has been proposed for preserving the assumed properties when the structured (elemental) parameter uncertainties are added into the linear discrete singular time-delay system. When all the finite eigenvalues lie inside the unit circle of the z-plane, the proposed criterion will become the stability robustness criterion. By mathematical analysis, the proposed criterion has been proved to be less conservative than that presented by Xu and Lam [3].

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